

# Non-linear analysis in RL-LED optoelectronic circuit

M. P. HANIAS\*, L. MAGAFAS<sup>a</sup>, J. KALOMIROS<sup>b</sup>

TEI of Chalkis, GR 34400, Evia, Chalkis

<sup>a</sup>Department of Electrical Engineering, Kavala Institute of Technology, St. Loukas 65404 Kavala, Hellas

<sup>b</sup>Technological and Educational Institute of Serres, Department of Informatics and Communications, GR 62100, Serres, Hellas

In this paper we present the chaotic state of a LED as a diode element in a non linear LRD. Multisim is used to simulate the circuit and show the presence of chaos. Time series analysis performed by the method proposed by Grassberger and Procaccia. The correlation and minimum embedding dimension  $v$  and  $m$  respectively were calculated. Also the corresponding Kolmogorov entropy was calculated.

(Received January 7, 2008; accepted February 7, 2008)

Keywords: Signal processing, Optoelectronic, Chaos, Non- linear analysis, LED

## 1. Introduction

The research direction of dynamical chaos is gradually moving towards practical applications and there is a growing interest for chaotic signal generation sources. In regard to this, various circuits have been proposed. In a recent paper [1] we had study the chaotic RLD circuit We used a commercial diode type 1N4001GP as non linear element to produce chaos. Here we try to produce chaos using a common LED in its operation point. The complete circuit is very simple and its software simulated operation demonstrates how chaos can be generated. We use MultiSim as the appropriate circuit simulation software since it is known to provide an interface adequately close to real implementation [2,3]. The rest of the paper is organized as following. In Section 2, we describe the considered circuit and discuss on its operation basics towards chaos when driven by a sinusoidal input signal. The corresponding time series analysis is presented in Section 3. Final comments are outlined in Section 4.

## 2. Experimental

A non autonomous chaotic circuit driven RL-LED circuit shown in Fig 1

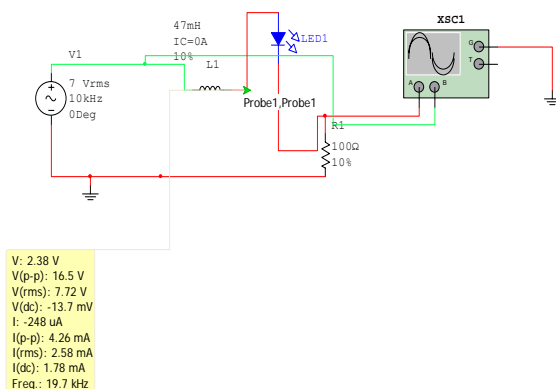


Fig. 1. RL-LED chaotic circuit in Multisim circuits simulation software.

It consists of a series connection of an ac-voltage source, a linear resistor  $R_1$ , a linear inductor  $L_1$  and a typical LED.

The  $R_1$  is  $R_1 = 100 \Omega$  in series with the LED. The circuit is driven by an input sinusoidal voltage with amplitude  $V_1$  as applied through an inductor  $L_1 = 47 \text{ mH}$ . The simulated circuit operation is monitored by checking the voltage across resistor  $R_1$ . Fig.2 shows the simulation obtained chaotic time series of the output signal for input signal amplitude  $V_{\text{rms}} = 7$  volts and frequency  $f = 10 \text{ KHz}$ .

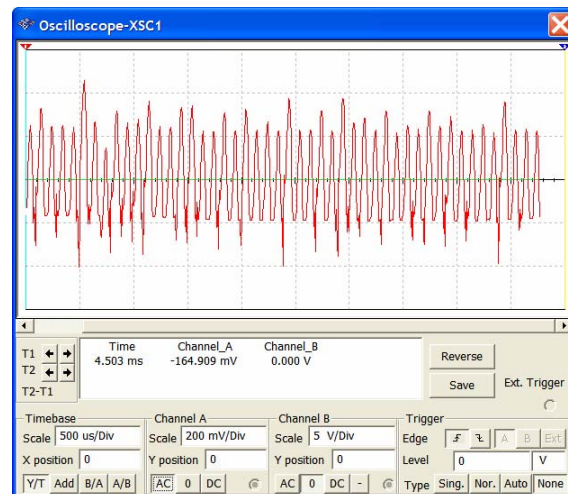


Fig. 2. Output chaotic signal  $V = V(t)$  across resistor  $R_1$  for the RL-LED circuit of Fig. 1

## 3. Time series analysis

We now proceed to the analysis of the obtained chaotic time series following the method proposed by Grassberger and Procaccia [4,5] and successfully applied in similar cases [6-9]. Moreover, according to Takens theory [10-11], the measured time series can be used to reconstruct the original phase space. At first, we calculate

the correlation integral  $C(r)$  for the simulated output signal for  $\lim r \rightarrow 0$  and  $N \rightarrow \infty$ , generally as defined by [12]:

$$C(r) = \frac{1}{N_{pairs}} \sum_{\substack{l=1, \\ j=l+W}}^N H(r - \|\vec{X}_l - \vec{X}_j\|) \quad (1)$$

where  $N$  is the number of the corresponding time series points,  $W$  is the Theiler window [12],  $H$  is the Heaviside function [12], and

$$N_{pairs} = \frac{2}{(N-m+1)(N-m+W+1)} \quad (2)$$

with  $m$  being the embedding dimension. Clearly, the summation in eq.(1) counts the number of pairs  $(\vec{X}_l, \vec{X}_j)$  for which the distance, i.e. the Euclidean norm,  $\|\vec{X}_l - \vec{X}_j\|$  is less than  $r$  in an  $m$  dimensional Euclidean space. Here, the number of the experimental points is  $N=10896$ , while considering the  $m$  dimensional space, each vector  $\vec{X}_l$  will be given by [14]:

$$\vec{X}_l = \{V(t_i), V(t_i + \tau_d), V(t_i + 2\tau_d), \dots, V[t_i + (m-1)\tau_d]\} \quad (3)$$

and represent a point of the  $m$  dimensional phase space in which the attractor is embedded each time. In eq.(3),  $\tau_d$  is the time delay determined by the first minimum of the mutual information function  $I(\tau_d)$  and defined as  $\tau_d = \ell \Delta t$  with  $\ell=1, 2, \dots, N$  and  $\Delta t=6.25\mu s$  is the sample rate. As shown in Fig.3, in our case the mutual information function  $I(\tau_d)$  exhibits a local minimum at  $\tau_d=5$  time steps and, thus, we shall consider  $\tau_d=5$  as the optimum delay time.

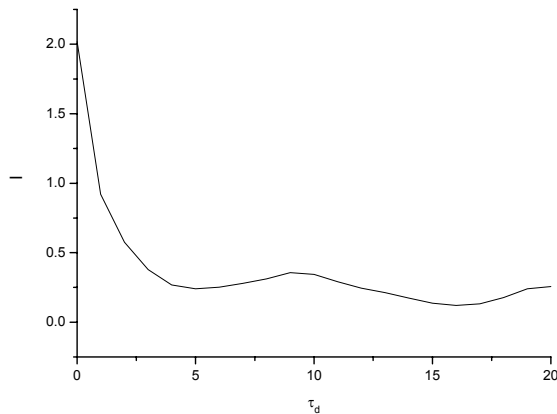


Fig. 3. Average Mutual Information  $I(\tau_d)$  vs. time delay  $\tau_d$ .

Next, we deal with parameter  $W$  which is the Theiler window. As Theiler pointed out if temporally correlated points are not neglected, spuriously low dimension estimate may be obtained [8-12]. However, since there is no concrete rule of how to choose  $W$ , it may take the first zero-crossing value of the correlation function  $CR(\tau_d)$ , as suggested by Kantz and Schreiber [8-12]. This means that we can use the correlation length as a starting value for  $W$  [12]. As shown in Fig.4, the correlation length  $\tau_c$  is equal to  $\tau_c=5$  and, thus,  $W = \tau_c=5$  time lags. Fig.4 also depicts a strong correlation between the data indicating the way past states affect the system's current state. Hence, we can use these values for phase space reconstruction.

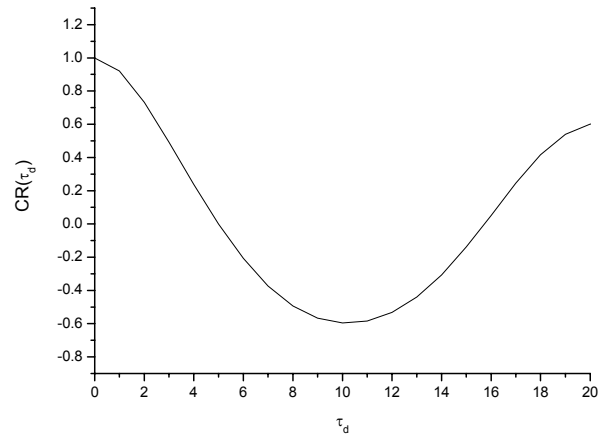


Fig. 4. Correlation function  $CR(\tau_d)$  vs. time delay  $\tau_d$ .

With eq.(1) dividing the considered  $m$  dimensional phase space into hypercubes with a linear dimension  $r$ , we count all points with mutual distances less than  $r$ . Then, it has been proven [15-18] that if the attractor is a strange one, the correlation integral will be proportional to  $r^\nu$ , where  $\nu$  is a measure of the attractor's dimension called correlation dimension. By definition, the correlation integral  $C(r)$  is the limit of correlation sum of eq.(1) and is numerically calculated as a function of  $r$  from eq.(1) for embedding dimensions  $m=1, \dots, 10$ . Fig.5 depicts the relation between the logarithms of correlation integral  $C(r)$  and  $r$  for different embedding dimensions  $m$ . As seen in Fig.6, the slopes  $\nu$  of the lower linear parts of these log-log curves provide all necessary information for characterizing the attractor. Then, in Fig.7, the corresponding average slopes  $\nu$  are given as a function of the embedding dimension  $m$  indicating that for high values of  $m$ ,  $\nu$  tends to saturate at the non integer value of  $\nu=2.23$ . For this value of  $\nu$ , the minimum embedding dimension can be  $m_{min}=3$  [12], and thus, the minimum embedding dimension

of the attractor for one to one embedding will be equal to 3.

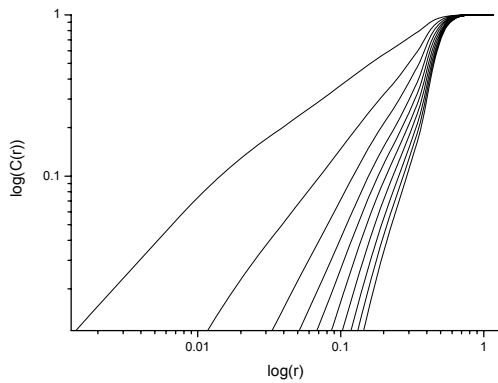


Fig. 5. Relation between  $\log C(r)$  and  $\log r$  for different embedding dimensions  $m$ .

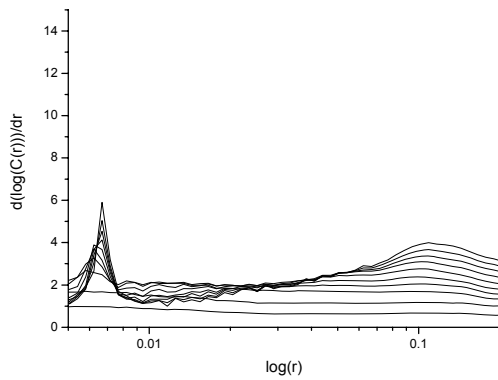


Fig. 6. The corresponding slopes and scaling region of Fig.5.

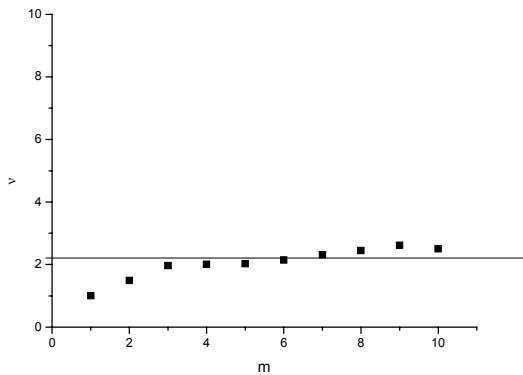


Fig. 7. Correlation dimension  $\nu$  vs. embedding dimension  $m$ .

Following the above and in order to get accurate measurements of the strength of the chaos present in the

oscillations of the simulated output signal, we introduce the Kolmogorov entropy. According to [12], the method followed so far also leads to an estimate of the Kolmogorov entropy, i.e. the correlation integral  $C(r)$  scales with the embedding dimension  $m$ , since:

$$C(r) \sim e^{-m\tau_d K_2} \quad (4)$$

where  $K_2$  is a lower bound to the Kolomogorov entropy. Fig.8 shows the relation between  $K_2$  and the logarithm of  $r$  for different embedding dimensions  $m$ , while the plateau, indicates that  $K_2=0.52$  bit/s, meaning that there is a steady lose of information at a constant rate given by  $K_2$ .

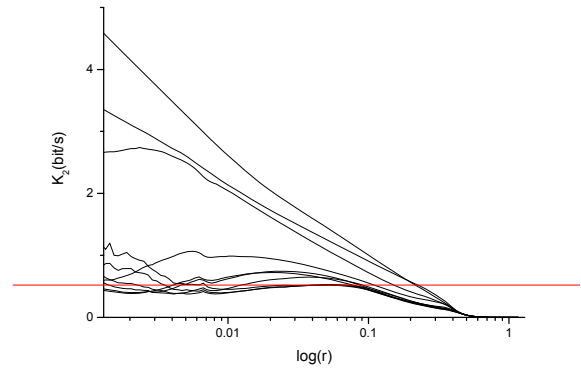


Fig. 8. Kolmogorov entropy vs.  $\log r$  for embedding dimensions  $m=2, \dots, 10$ .

## 5. Conclusion

The scaling behaviour of the correlation integral and the saturation of correlation dimension  $\nu$  with increasing embedding dimensions  $m$  reflect low dimensionality. The strange attractor that governs the phenomenon has a correlation dimension  $\nu=2.23$  stretching and folding in a 3 dimension phase space. Thus, the number of degrees of freedom of the whole domain structure is limited at 3 and this results in the low value of the correlation dimension. The LED exposes chaotic behaviour even if it works in its operation point. In this work, the obtained simulation results indicate that the proposed circuit can be used to generate chaotic signal, in a light emitting manner, useful in code and decode applications.

## References

- [1] M. P. Haniias, G. Giannaris, A. Spyridakis, A. Rigas, Chaos, Solitons & Fractals, **27**(2), 569 (2006).
- [2] K. E. Lonngren, IEEE Transactions on Education, **34**(1), February (1991).
- [3] G. Mykolaitis, A. Tamaševičius, S. Bumelienė, Electronics Letters, **40**, (2), 91 (2004).
- [4] P. Grassberger, I. Procaccia, Phys. Rev Lett. **50**, 346 (1983).

- [5] P. Grassberger, I. Procaccia, *Physica D* **9**, 189 (1983).
- [6] M. P. Haniyas, J. A. Kalomiros, Ch. Karakotsou, A. N. Anagnostopoulos, J. Spyridelis, *Phys. Rev B* **49**, 16994 (1994).
- [7] M. P. Haniyas, J. A. N. Anagnostopoulos, *Phys. Rev B* **47**, 4261 (1993).
- [8] P. Stelter, *Chaos, Solitons & Fractals* **1**(3), 273 (1991).
- [9] Alfonso Bueno-Orovio, Victor M. Perez-Garcia, *Chaos, Solitons & Fractals*, **34**, 509 (2007).
- [10] F. Takens, *Lecture Notes in Mathematics*, 898, (1981).
- [11] Jiang Wang, Li Sun, X. Fei, B. Zhu, *Chaos, Solitons & Fractals* **33**(3), 901 (2007).
- [12] H. Kantz, T. Schreiber "Nonlinear Time Series Analysis", Cambridge University Press, Cambridge (1997).

---

\*Corresponding author: [mhanias@teihal.gr](mailto:mhanias@teihal.gr)